Bの効用関数　$u = 2X^2 + 2XY + 2Y^2$

$$MU_X = 4X + 2Y$$
$$MU_Y = 4Y + 2X$$
$$MRS = \frac{4X+2Y}{4Y+2X} = \frac{2X+Y}{2Y+X}$$

効用最大化の一階の条件(F.O.C.)

$$M = P_X \cdot X^* + P_Y \cdot Y^* \quad \cdots ①$$
$$\frac{2X^*+Y^*}{2Y^*+X^*} = \frac{P_X}{P_Y} \quad \cdots ②$$

②より　$P_Y(2X^* + Y^*) = P_X(2Y^* + X^*)$

$$2P_Y X^* + P_Y Y^* = 2P_X Y^* + P_X X^*$$
$$(2P_Y - P_X)X^* = (2P_X - P_Y)Y^*$$
$$Y^* = \frac{2P_Y - P_X}{2P_X - P_Y} X^* \quad \cdots ③$$

③を①に代入

$$M = P_X \cdot X^* + P_Y\left(\frac{2P_Y - P_X}{2P_X - P_Y}\right)X^*$$
$$= X^*\left\{P_X + P_Y\left(\frac{2P_Y - P_X}{2P_X - P_Y}\right)\right\} = X^*\left(\frac{2P_X^2 - P_XP_Y + 2P_Y^2 - P_X}{2P_X - P_Y}\right.$$
$$= X^*\left(\frac{2P_X^2 - 2P_XP_Y + 2P_Y^2}{2P_X - P_Y}\right)$$

$$\boxed{X^* = \frac{M(2P_X - P_Y)}{2P_X^2 - 2P_XP_Y + 2P_Y^2}} \quad \cdots ④$$

← Xに対する需要関数.

x^*の　価格弾力性 を求める

$$\eta_d = \frac{\partial x^*}{\partial P_x} \cdot \frac{P_x}{x^*}$$ なので $\frac{\partial x^*}{\partial P_x}$ をまず求める

$$\frac{\partial x^*}{\partial P_x} = \frac{M}{2}\left\{ \frac{2}{P_x^2 - P_x P_Y + P_Y^2} - \frac{(2P_x - P_Y)(2P_x - P_Y)}{(P_x^2 - P_x P_Y + P_Y^2)^2} \right.$$

$$= \frac{M}{2}\left\{ \frac{2(P_x^2 - P_x P_Y + P_Y^2) - (2P_x - P_Y)^2}{(P_x^2 - P_x P_Y + P_Y^2)^2} \right\}$$

$$\eta_d = \frac{M}{2}\left\{ \frac{2P_x^2 - 2P_x P_Y + 2P_Y^2 - (2P_x - P_Y)^2}{(P_x^2 - P_x P_Y + P_Y^2)^2} \right\} \frac{P_x}{x^*} \quad \cdots ⑤$$

⑤に④を代入し

$$\eta_d = \frac{\cancel{M}}{\cancel{2}}\left\{ \frac{2P_x^2 - 2P_x P_Y + 2P_Y^2 - (2P_x - P_Y)^2}{(P_x^2 - P_x P_Y + P_Y^2)^{\cancel{2}}} \right\} \frac{P_x \cdot \cancel{2}\cancel{(P_x^2 - P_x P_Y + P_Y^2)}}{\cancel{M}(2P_x - P_Y)}$$

$$\boxed{\eta_d = \frac{P_x\{2P_x^2 - 2P_x P_Y + 2P_Y - (2P_x - P_Y)^2\}}{(2P_x - P_Y)(P_x^2 - P_x P_Y + P_Y^2)}}$$ ⇐ 価格弾力性.

交差弾力性 : $\eta_c = \frac{\partial x^*}{P_Y} \cdot \frac{P_Y}{x^*}$

$$\frac{\partial x^*}{\partial P_Y} = \frac{M}{2}\left\{ \frac{-1}{(P_x^2 - P_x P_Y + P_Y^2)} - \frac{(2P_x - P_Y)(2P_Y - 2P_x)}{(P_x^2 - P_x P_Y + P_Y^2)^2} \right\}$$

$$= \frac{M}{2}\left\{ \frac{-(P_x^2 - P_x P_Y + P_Y^2) - 2(2P_x - P_Y)(P_Y - P_x)}{(P_x^2 - P_x P_Y + P_Y^2)^2} \right\}$$

$$\eta_c = \frac{\partial x^*}{\partial P_Y}\frac{P_Y}{x^*} = \frac{\cancel{M}}{\cancel{2}}\left\{\frac{-(P_X^2 - P_X P_Y + P_Y^2) - 2(2P_X - P_Y)(P_Y - P_X)}{(P_X^2 - P_X P_Y + P_Y^2)^2}\right\} \times \frac{P_X(\cancel{2}P_X^2 - 2P_X P_Y + 2P_Y^2)}{\cancel{M}(2P_X - P_Y)}$$

$$\eta_c = \frac{-P_X((P_X^2 - P_X P_Y + P_Y^2) - 2(2P_X - P_Y)(P_Y - P_X))}{(P_X^2 - P_X P_Y + P_Y^2)(2P_X - P_Y)}$$

所得弾力性

$$\eta_M = \frac{\partial x^*}{\partial M}\frac{M}{x^*}$$

$$\frac{\partial x^*}{\partial M} = \frac{2P_X - P_Y}{2(P_X^2 - P_X P_Y + 2P_X^2)}$$

$$\eta_M = \frac{\partial x^*}{\partial M}\cdot\frac{M}{x^*} = \frac{\cancel{(2P_X - P_Y)}}{\cancel{2}(P_X^2 \cancel{- P_X P_Y + P_Y^2})}\cdot\frac{\cancel{2M}(\cancel{P_X^2 - P_X P_Y} + P_Y^2)}{\cancel{M}(\cancel{2P_X - P_Y})}$$

$$\eta_M = 1$$